$$u_t = a\cdot u\left(1-\frac{u}{u_0}\right)$$

$$u(0) = u_0/2$$

Найти t^*: $u(t^*) = 0,99u_0$;

$$\frac{du}{dt} = au\left(1-\frac{u}{u_0}\right) \Leftrightarrow$$

$$\frac{du}{u\left(1-\frac{u}{u_0}\right)} = a\,dt \Leftrightarrow$$

$$\int\frac{du}{u\left(1-\frac{u}{u_0}\right)} = \left[\frac{A}{u} + \frac{B}{1-\frac{u}{u_0}} = \frac{u\left(B-\frac{A}{u_0}\right)+A}{u\ldots}\right] =$$

$$= \left.\begin{matrix}A = 1\\ B = \frac{1}{u_0}\end{matrix}\right\} = \int\frac{du}{u} + \frac{1}{u_0}\int\frac{du}{1-\frac{u}{u_0}} =$$

$$= \ln|u| - \ln|u_0 - u| = at + C \Rightarrow$$

$$\Rightarrow \frac{\cancel{\ln|u/(u_0 - u)|}}{a} = t \Rightarrow$$

$$t^* = \frac{\cancel{\ln|100|}}{a}$$

подставить н/у $u(0) = u_0/2$ найти C

подставить $u(t^*) = 0,99u_0$

№1 $u_{xx} + x\,u_{yy} = f(x,y,u)$ (1)

$-x < 0 \quad x \in (0,+\infty)$

$x = 0 \quad x \in \{0\}$

$-x > 0 \quad x \in (-\infty, 0)$ - област гиперб-ти

$dy^2 + x\,dx^2 = 0 \qquad dy = \pm\sqrt{-x}\,dx$

на $(-\infty, 0) \Rightarrow \quad y = \pm \frac{2}{3}\sqrt{-x^3} + C \Rightarrow$

$y \pm \frac{2}{3}\sqrt{-x^3} = C$ (характеристика (1))

$\xi = y + \frac{2}{3}\sqrt{-x^3} \qquad \eta = y - \frac{2}{3}\sqrt{-x^3}$

$u_x = u_\xi \xi_x + u_\eta \eta_x = u_\xi(-\sqrt{-x}) + u_\eta(\sqrt{-x})$

$u_y = u_\xi \xi_y + u_\eta \eta_y = u_\xi + u_\eta$

$u_{xx} = u_{\xi\xi}(\xi_x)^2 + 2u_{\xi\eta}\xi_x\eta_x + u_{\eta\eta}(\eta_x)^2 + u_\xi \xi_{xx} + u_\eta \eta_{xx}$

~~$u_{yy} = u_\xi = u_{\xi\xi}\frac{1}{2\sqrt{-x}} - 2u_{\xi\eta}$~~

$-u_{\xi\xi}x + 2x\,u_{\xi\eta} - x\,u_{\eta\eta} + u_\xi \frac{1}{2\sqrt{-x}} - \frac{u_\eta}{2\sqrt{-x}}$

$u_{yy} = u_{\xi\xi} + 2u_{\xi\eta} + u_{\eta\eta}$

подставим в (1):

~~u_{xx}=~~

$-\cancel{u_{\xi\xi}x} + 2x\,u_{\xi\eta} - \cancel{x\,u_{\eta\eta}} + \cancel{u_{\xi\xi}x} + 2x\,u_{\xi\eta}$

$+ \cancel{x\,u_{\eta\eta}} \ \ldots \Rightarrow$

$$\Rightarrow u_{\xi\eta} = \frac{f\left(\left(\frac{3}{4}(\xi - \eta)\right)^{\frac{2}{3}}, \frac{\eta+\xi}{2}, u\right) + u_\xi\frac{1}{2\sqrt{-x}} - \frac{u_\eta}{2\sqrt{-x}}}{4x}$$

№2 $u_{xx} - y u_{yy} = f(x, y, u)$ (1)

$y < 0$ $y \in (-\infty, 0)$ область эл-ти

$y = 0$ $y \in \{0\}$ об — " — П

$y > 0$ $y \in (0, +\infty)$ — " — Г

$dy^2 - y\,dx^2 = 0 \Leftrightarrow dy = \pm i \sqrt{y}\,dx$

$\Rightarrow \frac{dy}{\sqrt{y}} = \pm i\,dx \Rightarrow 2 \cdot \sqrt{y} = \pm i x + C$

$\Rightarrow 2 \cdot \sqrt{y} \pm i x = C$

$\Rightarrow \xi = 2 \cdot \sqrt{y} \quad \eta = x$

$u_{xx} = u_{\xi\xi}(\xi_x)^2 + 2u_{\xi\eta}\xi_x\eta_x + u_{\eta\eta}(\eta_x)^2 + u_\eta \eta_{xx} + u_\xi \xi_{xx}$

$= 0 + 0 + u_{\eta\eta} + 0 + 0$

$u_{yy} = \frac{u_{\xi\xi}}{y} + 0 + 0 + u_\xi\left(-\frac{1}{2} y^{-3/2}\right) + 0$

подставим в (1)

$u_{\eta\eta} + \frac{u_{\xi\xi}}{y} \cdot y - \frac{1}{2}\frac{u_\xi}{\sqrt{y^3}} \cdot y = f(\eta, \frac{\xi^2}{4}, u)$

$$\boxed{u_{\eta\eta} + u_{\xi\xi} = \frac{1}{2}\frac{u_\xi}{\sqrt{y}} + f(\eta, \frac{\xi^2}{4}, u)}$$

$$\frac{\;\;+\xi}{2}, u\Big) + u_\xi \frac{1}{2\sqrt{x}} - \frac{u_\eta}{2\sqrt{x}}$$

$$4x$$

$$T' = -\lambda T \qquad T = C_1 e^{-\lambda t}$$

$$\cancel{T(0) = 1} \qquad \cancel{T(0) = 1 \Rightarrow C = 1}$$

$$1 = \sum_{n=1}^{\infty} C_n \sin \frac{\pi n}{l} x$$

$$C_n = \frac{2}{l} \int_0^l 1 \cdot \sin\left(\frac{\pi n}{l} x\right) dx = -\frac{2}{\pi n}(-1 + \cos \pi n)$$

$$u = \sum_{n=1}^{\infty} C_n \cdot e^{-\left(\frac{\pi n}{l}\right)^2} \sin \frac{\pi n}{l} x$$

$\sin\left(\frac{\pi n}{l}\right) x$

№ 7

$$l_1 = 2 l_2 = 3 l_3$$

$$\boxed{\nu_3 = \pi^2 \left(\frac{1}{l_1^2} + \frac{4}{l_1^2} + \frac{9}{l_1^2} \right) a^2} =$$

$$\Rightarrow \nu_3 = \pi^2 \frac{14}{l_1^2} a^2 \Rightarrow l_1 = \sqrt{\frac{14}{\nu_3}} \pi a$$

$$\left.\begin{aligned} V_{\square} &= \frac{14 \cdot 6 \cdot \sqrt{14}}{\nu_3^{3/2}} \pi^3 a^3 \\ \div \\ V_{\square} &= \frac{3\sqrt{3}\, \pi^3 a^3}{\nu_3^{3/2}} \end{aligned}\right\} = 28 \sqrt{\frac{14}{3}} \text{ раза больше}$$

.192 $u_{xx} - y u_{yy} = f(x, y, u)$ (1)

$u_{xx} - y u_{yy} = f(x, y, u)$

$y < 0$ $y \in (-\infty, 0)$ область гипербол.

$y = 0$ $y \in \{0\}$ область парабол.

$y > 0$ $y \in (0, +\infty)$ область эллиптич.

$(dy)^2 - y(dx)^2 = 0$ $\quad$ ~~$dy = i\sqrt{|x|}\,dx$~~

$dy = i\sqrt{|x|}\,dx$ $\quad$ или $dy = i\sqrt{x}\,dy$ $x \in (0, +\infty)$

$y = i\sqrt{x^3}$ $\qquad$ $y - \frac{i2}{3}\sqrt{x^3} = C$

$\xi = y$ $\qquad \eta = -\frac{2}{3}\sqrt{x^3}$

$u_{xx} = u_{\xi\xi}(\xi_x)^2 + 2u_{\xi\eta}\xi_x\eta_x + u_{\eta\eta}(\eta_x)^2$

$+ u_\xi \xi_{xx} + u_\eta \eta_{xx} =$

$= u_{\xi\xi}\cdot 0 + 0 + u_{\eta\eta}\cdot x + 0 - \frac{u_\eta}{2\sqrt{x}}$

$u_{yy} = 0 + 0 + u_{\xi\xi} + u_\xi \underset{0}{\underset{\parallel}{\xi_{yy}}} + u_\eta \underset{0}{\underset{\parallel}{\eta_{yy}}}$

$= u_{\xi\xi}$

$\left(\ \frac{\ }{2},\ u\right) + u_\xi \frac{1}{2\sqrt{x}} - \frac{u_\eta}{2\sqrt{x}}$

$4x$

№5

$u_t = u_{xx} \quad x \in (0, l),\ t > 0$

$u(x, 0) = 1$

$u(0, t) = u(l, t) = 0$

Пусть $u = XT \qquad XT' = X''T \ \Big|\, : XT$

$\Rightarrow \quad \frac{T'}{T} = \frac{X''}{X} = -\lambda$

решим ~~кр~~ задачу Штурма-Лиувилля

$$\begin{cases} X'' + \lambda X = 0 \\ \cancel{u(0,t)} = X(0) = X(l) = 0 \end{cases}$$

Пусть $\lambda < 0 \Rightarrow X'' = \lambda X \Rightarrow \mu = \pm\sqrt{\lambda}$

$\Rightarrow \quad X = C_1 e^{\sqrt{\lambda} x} + C_2 e^{-\sqrt{\lambda} x}$

$$\left.\begin{aligned} X(0) &= C_1 + C_2 = 0 \\ X(l) &= C_1 e^{\sqrt{\lambda} l} + C_2 e^{-\sqrt{\lambda} l} = 0 \end{aligned}\right\} \Rightarrow C_1 = C_2 = 0$$

Пусть $\lambda = 0 \quad X = C_1 x + C \Rightarrow C_1 = C_2 = 0$

Пусть $\lambda > 0 \qquad X = C_1 \sin \lambda x + C_2 \cos \lambda x$

$X(0) = C_2 = 0$

$X(l) = C_1 \sin \lambda l = 0 \Rightarrow \lambda_n = \left(\frac{\pi n}{l}\right)^2$

$$u = \sum_{n=1}^{\infty} T_n \sin\left(\frac{\pi n}{l}\right) x = \sum_{n=1}^{\infty} C_n e^{-\left(\frac{\pi n}{l}\right)^2 t} \cdot \sin\left(\frac{\pi n}{l}\right) x$$

№4. $u_t = u_{xx}$ $\quad x \in \mathbb{R} \quad t>0$

$$u(x,0) = \begin{cases} 1 & |x|<1 \\ 0 & |x|>1 \end{cases}$$

u - огр

решение даётся в виде интеграла Пуассона

$$u(x,t) = \frac{1}{2a\sqrt{\pi t}} \int_{-\infty}^{\infty} \exp\left(-\frac{(\xi - x)^2}{4a^2 t}\right) \varphi(\xi)\, d\xi$$

где $a = 1$ $\quad \varphi(x) = u(x,0)$

применим:

$$u(x,t) = \frac{1}{2\sqrt{\pi t}} \int_{-1}^{1} e^{\left(\frac{(\xi - x)^2}{4t}\right)} \cdot 1\, d\xi = \cancel{[\xi - x = \eta]} =$$

$$= \frac{1}{2\sqrt{\pi t}} \int_{-1}^{1} e^{\left(-\frac{\eta^2}{4t}\right)} d\eta = \left[\frac{\eta}{\sqrt{4t}} = \psi\right] =$$

$$= \frac{2}{\sqrt{\pi}} \int_{-1}^{1} e^{-\psi^2} d\psi$$

$\frac{2}{4x}$, $u) + u_1 \frac{1}{2\sqrt{x}} = \frac{u_1}{2\sqrt{x}}$

№3.

$u_{tt} = u_{xx}$ $\quad x \in (-\infty, \infty)$, $t > 0$

$u(x,0) = \sin\pi x$, $|x| < 1$, 0 $|x| > 1$.

$u_t(x,0) = 0$

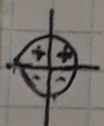
~~По~~ По формуле Даламбера.

при $|x| < 1$:

$u(x,t) = \frac{\sin\pi(x-t) + \sin\pi(x+t)}{2} + 0 \Rightarrow$

при $|x| > 1$ $\quad u(x,t) \equiv 0$

~~$u(x,t) = \sin\pi$~~

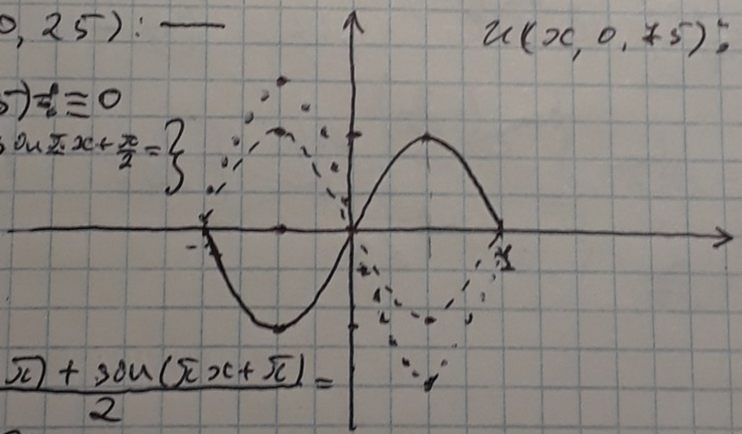


$u(x, 0,25)$: ——

$u(x, 0,75)$: - - -

$u(x, 0,5) \equiv 0$

$\left\{ \sin(\pi x - \frac{\pi}{2}) + \sin(\pi x + \frac{\pi}{2}) = \right.$
$\left. = \cancel{\cos\pi x}\ 0 \right\}$



$u(x, 1)$:

$\frac{\sin(\pi x - \pi) + \sin(\pi x + \pi)}{2} =$

$= \sin\pi x$